Pacific Structural Steel Conference 2007
Steel Structures in Natural Hazards
Wairakei, New Zealand
13-16 March 2007

EVALUATION OF LAY'S EQUATION FOR PLASTIC HINGE LOCATION IN STEEL COLUMNS

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ABSTRACT

The current New Zealand steel structures standard has a provision that aims to restrict plastic hinges to develop only at the member ends. The equations in this provision were adopted from Lay's PhD thesis which were developed not only to encourage yielding at the member end, but also to ensure sufficient member rotational capacity can be maintained. The accuracy of the end yielding criteria in Lay's equations is examined using an analytical technique developed in this study. The analytical procedure incorporates non-linear geometric and material effects by considering stability functions in conjunction with residual stress effects. It is found that the code equations are more conservative compared with the analysis results. A new equation to prevent yielding away from the member ends is proposed in the research project. This forms the basis for the proposed amendments to Clause 8.4.3.2 in New Zealand steel structures standard.

Introduction

The New Zealand and Australian steel codes (SNZ 1997, SAA 1975) have a provision for the location of plastic hinges in columns that is not present in the codes of other countries. It aims to restrict the plastic hinges to form only at the column ends. This is a more desirable behaviour than yielding along the member length because the region beside the plastic hinges can be effectively braced against local and lateral buckling to ensure sufficient inelastic rotational capacity. It also enables the true collapse mechanism and inelastic rotational demands to be calculated more easily.

Clause 8.4.3.2 of New Zealand steel structures standard, NZS 3404 (SNZ 1997), specifies that a member with a slenderness limit, λ, end moment ratio, β, and axial force ratio, $N^*/\phi N_S$, must satisfy Eqs. 1 and 2, where ϕ is the resistance factor, N^* is the applied axial force and N_S is the nominal section axial capacity, to ensure plastic hinges would form at the member ends. These equations were developed by Lay (Lay 1964) to ensure columns with a sufficient rotational deformation capacity are used. Different approaches were used to derive these equations. For axial force ratios greater than 0.15, Eq. 1 was adopted by curve fitting column deflection curve data in Lay's thesis for high axial loaded members. For axial force ratios less or equal to 0.15, Eq. 2 was developed using elastic stability formulation with the maximum moment in the member being limited to less than 1.05 times the design moment at the column ends. Note that with the value of 1.05, it implies that the maximum moment is permitted to move away from the member ends. Lay argues that as the axial force on the member is small, the effect of plastic hinges occurring away from the member ends is not

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likely to be detrimental.

$$\frac{N^*}{\phi N_S} \leq \left[\frac{1+\beta-\lambda}{1+\beta+\lambda}\right] \qquad \text{when} \qquad \frac{N^*}{\phi N_S} > 0.15 \tag{1}$$

$$\frac{N^*}{\phi N_S} \leq \left[\frac{0.6+0.4\beta}{\lambda}\right]^2 \qquad \text{when} \qquad \frac{N^*}{\phi N_S} \leq 0.15 \tag{2}$$

The background to Lay's equations are not clearly described in his thesis; there is a discontinuity at $N^*/\phi N_S$ = 0.15 as shown in Fig. 1; and the equations often govern the sizes of members in seismic frame design. Therefore, there is a need to re-evaluate the provisions in NZS 3404 for end yielding criteria, EYC. This paper describes the research carried out at the University of Canterbury to evaluate the accuracy of the end yielding criteria in Lay's equations.

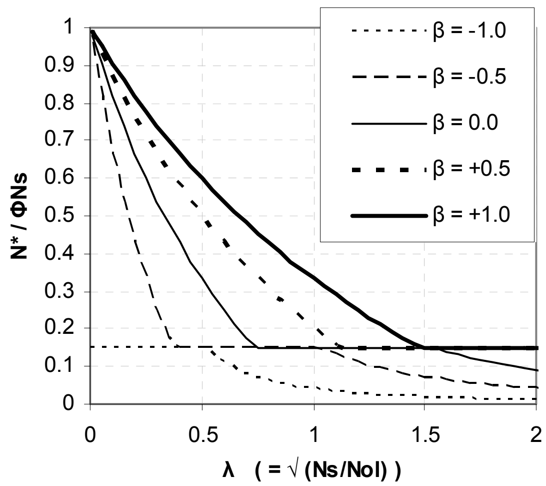


Figure 1. Axial force limits for different member slenderness and end moment ratio (SAA 1975).

Analytical Model Development

The analytical model was developed based on stability functions which consider reduction in flexural stiffness from axial forces due to the geometric nonlinearity. It is also based on the New Zealand column design curves which consider the reduction in flexural stiffness due to material non-linearity that arises from initial residual stresses effects, member out-of-straightness and accidental non-concentric loading. The model was developed using the computer program "MATLAB" (Matlab 2005). A brief description of the analysis procedure used in this project is summarized below. Detailed description on the theory behind stability functions and effectiveness stiffness approaches are described in the research report (Peng et al. 2006).

Stability functions were used to derive a global stiffness matrix that relates lateral displacements and rotations (v_A, v_B, v_C, θ_A, θ_B, θ_C) to the shear forces and moments $(V_A,\ V_B,\ V_C,\ M_A,\ M_B,\ M_C)$ at nodes A, B and C for the column as shown in Fig. 2 under an axial compression force, *N*. The reduction in flexural stiffness, $(EI)_t$, due to residual stress effect was calculated by taking the ratio between the effective lengths corresponding to the inelastic column buckling curves and elastic Euler buckling curves, $(kL)_t$ and $(kL)_e$ as shown in Eq. 3. Note that the inelastic column buckling curves in NZS 3404 were developed not only considering the initial residual stress effect but also the effect of accidental non-concentric loading and member out-of-straightness. Consequently the elastic flexural stiffness obtained in this study is slightly conservative.

$$(EI)_t = EI\left(\frac{(kL)_t}{(kL)_e}\right)^2 \tag{3}$$

where $(kL)_t = \sqrt{\pi^2 (EI)_t / N}$, $(kL)_e = \sqrt{\pi^2 EI/N}$, E is Young's modulus and I is second moment of area of the section.

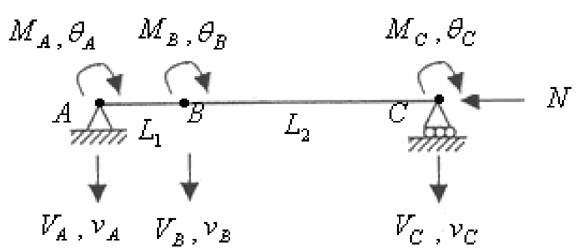


Figure 2. Beam-column member containing 2 sub-members and 2 internal degrees of freedom.

The overall analysis procedure to determine the end yielding criteria is an iterative procedure that requires the flexural stiffness being updated as the axial force is increased. The procedure is illustrated in Fig. 3. It may be seen that after a member with specific section properties was chosen, a small axial force, N, was applied and the effective flexural stiffness, $(EI)_t$, was calculated using Eq. 3. The stability functions with the effective flexural stiffness were then used to find the moment at node B, M_B, as illustrated in Fig. 2. If M_B is less than M_A, the axial force was gradually increased until M_B became greater than M_A. The axial force that caused the maximum moment to move towards node B corresponds to the critical axial force, N_C. Analyses were also carried out for different member slenderness, λ , and end moment ratios, β .

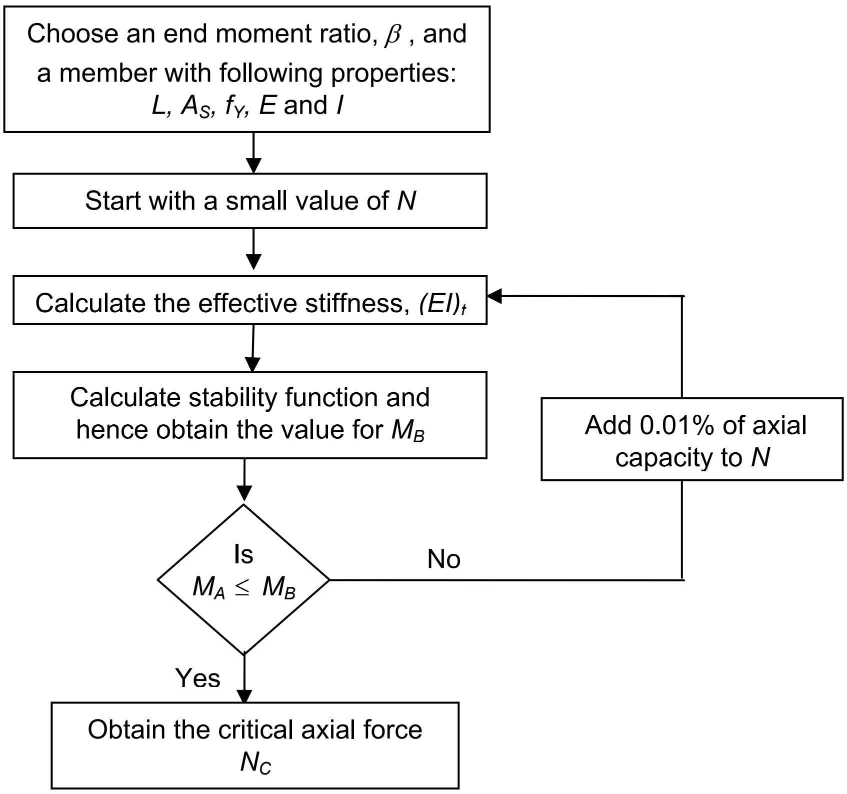


Figure 3. Overall analysis procedure for determining the critical axial force, N_C, for end yielding criteria.

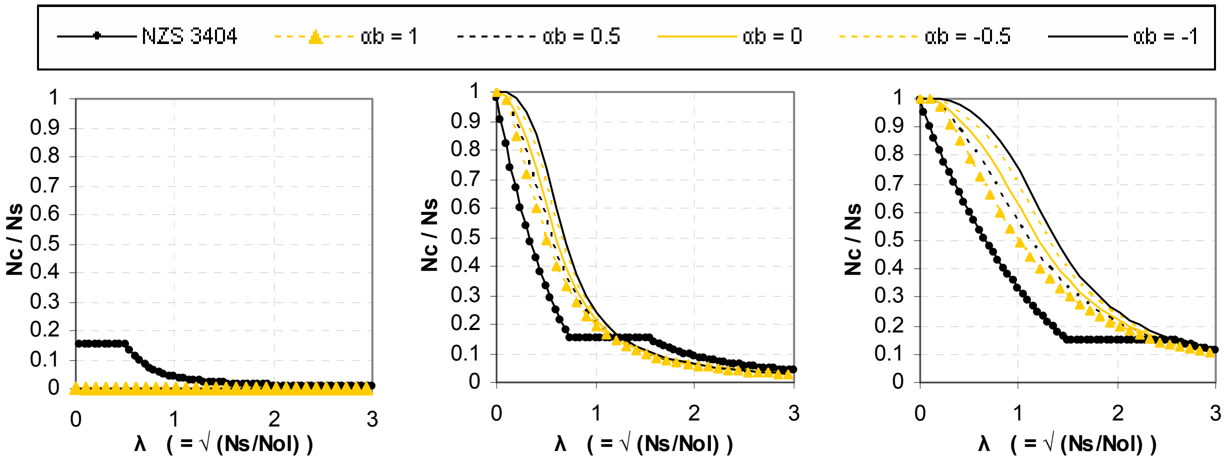
Analytical Predictions and Comparisons

The analytical axial force ratios that cause the maximum moment to move away from the member ends and existing provision in NZS 3404 are shown together in Fig. 4 for different end moment ratios, β . It may be seen that:

i) The provisions in NZS 3404 are generally conservative for columns with axial force ratios greater than 0.15.

ii) The provisions in NZS 3404 are non-conservative for columns with axial force ratios less than 0.15. This is expected as Lay's equations were developed by limiting the member moment to be 1.05 times the end moment for low axial loaded members.

iii) The end yielding curves from the analyses are continuous, and therefore more rational than the current provisions in NZS 3404, which have a discontinuity at an axial force ratio of 0.15.



(a) β = -1 (b) β = 0 (c) β = +1

Figure 4. Comparison of EYC curves from the analysis and the New Zealand steel code for different β

Proposed End Yielding Equations

As there is no closed form expression for the curves in Fig. 4, the empirical equation shown in Eq. 4 is developed for the end yielding criteria. This equation links the axial force ratio, N_C / N_S with the member slenderness limit, λ, and end moment ratio, β, in an exponential relationship. Three constants are required in this EYC equation which vary with different section types and residual stresses, α_b, as shown in Table 1.

$$\frac{N_C}{N_S} \le \left\{ \frac{A \times (\beta + 1)^B}{e^{\left(C/_{\beta+1} \right)}} \right\}^{\lambda} \tag{4}$$

where N_C is the nominal member capacity and N_S is the nominal section capacity.

Table 1. Coefficient for different section types

α_b	***A***	***B***	***C***
1	0.235	0.95	0.21
0.5	0.247	0.91	0.19
0	0.263	0.88	0.19
-0.5	0.265	0.92	0.17
-1	0.276	0.87	0.19

The proposed EYC equations are plotted together with the analysis results and the NZS 3404 provisions in Fig. 5. It may be seen that the proposed EYC equation is generally more conservative than the analysis results, especially for columns with axial force ratios higher than 0.5. As the end moment ratio, β, approaches 1, the analysis curves are harder to fit using an exponential function. Subsequently, the proposed EYC equation becomes more conservative for members with higher axial loads. However, the proposed equation is still much closer to the actual analysis results than the current provisions in NZS 3404. Adoption of the proposed EYC equation would relax the restriction on the column sizes specified in the current design guidelines.

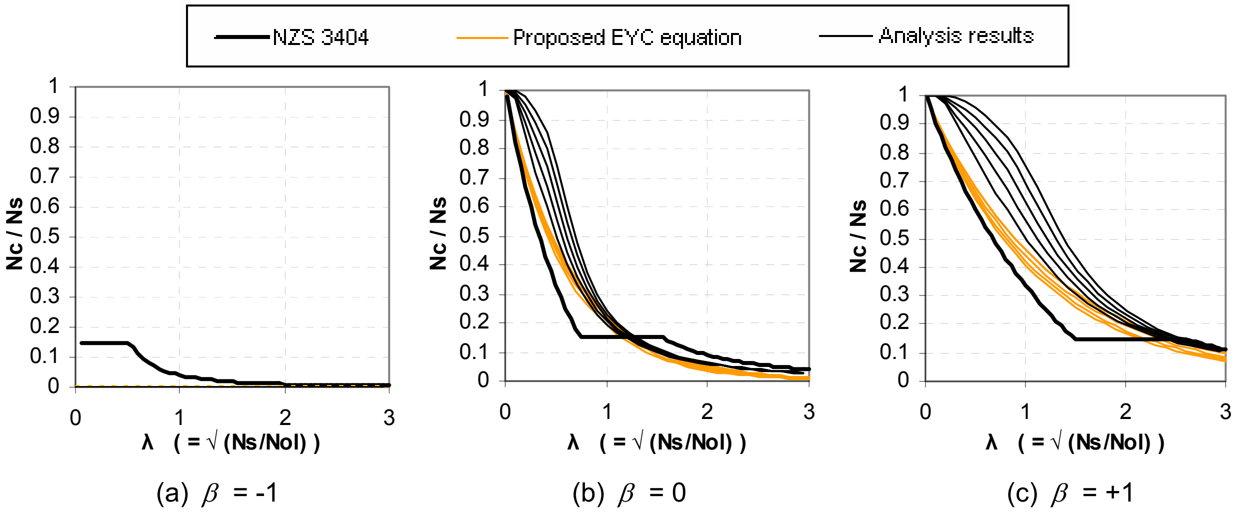


Figure 5. Comparison of NZS 3404 provision, analysis results and proposed EYC equation for different β

Conclusions

Based on the outcome of this study, the following conclusions can be drawn:

1/ Current code provisions in the New Zealand steel structures standard for end yielding criteria are generally found to be conservative for columns with axial force ratios higher than 0.15. However, for columns with lower axial forces, the provisions are found to be less conservative.

2/ New design equations are proposed in this study to represent the axial force ratio causing the plastic hinges to move away from the member ends. Unlike the current code equations, the proposed equations do not contain any discontinuity and match significantly better to the end yielding criteria curves from the analysis. These equations form the basis for the proposed amendments to Clause 8.4.3.2 in NZS 3404.

Acknowledgments

The authors are grateful for the financial support provided by the Steel Construction New Zealand, SCNZ, Heavy Engineering Research Association, HERA, and the University of Canterbury. All opinions expressed in this paper are those of the authors and do not necessarily represent those of the sponsors.

References

AISC. 2001. *Manual of Steel Construction, Load and Resistance Factor Design*, American Institute of Steel Construction, 2nd Ed., Chicago, USA.

Dr. Software. 1998. *Dr. Frame 2D*, Dr. Software, LLC, Seattle, USA.

Lay, M. G. 1964. *The Static Load Deformation Behaviour of Planar Steel Structures*, PhD Thesis, Lehigh University, Bethlehem, USA.

MATLAB, 2005, The MathWorks, Inc., Natick, MA, USA

Peng B. H. H., MacRae G. A., Walpole W. R., Moss P., Dhakal R. 2006. *Plastic Hinge Location in Columns of Steel Frames*, Civil Engineering Research Report, Department of Civil Engineering, University of Canterbury, NZ.

SAA. 1975. *SAA Steel Structures Code AS 1250*, Standards Association of Australia, Sydney, Australia.

SNZ. 1997. *Steel Structures Standard NZS 3404*, Standards New Zealand, Wellington, NZ.